MA5460 – Assignment 1
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1. Let n be the last two digits of your roll number. Verify the series Fourier expansion of the entry numbered $n \mod 20$ in Table 1 of the textbook.

2. Recall that D_n was the n-th Dirchlet kernel. Define

$$K_n := \frac{(D_0 + \cdots + D_n)}{n},$$

the Fejer kernel. Compute $K_n * f$.

3. Show that

$$K_n(x) = \frac{1}{2\pi n} \frac{\sin^2 nx/2}{\sin^2 x/2}.$$

4. Solve Problems 5 and 6 on Page 43 of the textbook.

5. Solve Problem 12 on Page 48 of the textbook.

6. Solve problems 7 and 8 on Page 213 and use these to prove Theorem 2.7.

7. Compute the Fourier transform of $f(x) = \frac{1}{x^2+a^2}, a > 0$ using the Calculus and residues and by using the inversion formula.

8. Show that the convergence of the Fourier series of f at a point x depends only on the behaviour of f near x, i.e., if $f(t) = g(t)$ for all t in some open interval containing x then the Fourier series of g converges to $g(x)$ at x if and only if the Fourier series of f converges to $f(x)$ at x.

9. (**The Schwartz Space**) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. We say f is *rapidly decreasing at infinity* if for each integer $m > 0$, the function $|x|^m f(x) \to 0$ as $x \to \infty$. Let S denote the collection of all infintely differentiable function all of whose derivatives are rapidly decreasing at infinity.

   a) Give several examples of functions in S.
   b) Show that S is an algebra over $\mathbb{R}$ with multiplication given by the usual product of functions.
   c) Show that the Fourier transform is well-defined on S and that $\hat{f} \in S$.
   d) Show that S is an algebra under the operation $*$.
   e) Let $f \in S$ and let $g = f + \hat{f} + \hat{\hat{f}} + \hat{\hat{\hat{f}}}$.

10. Show that

$$\int_0^\infty \frac{\sin rx}{x} dx = \int_{-\infty}^0 \frac{\sin rx}{x} dx = \frac{\pi}{2}.$$